Yoshihiro Tanaka[1] *and Katsutoshi Ohishi*[2]

Unified Approach to Optimization of Tread Pattern Shape and Cross-Sectional Contour of Tires[1]

REFERENCE: Tanaka, Yoshihiro and Ohishi, Katsutoshi, **"Unified Approach to Optimization of Tread Pattern Shape and Cross-Sectional Contour of Tires,"** *Tire Science and Technology*, TSTCA, Vol. 38, No. 4, October – December 2010, pp. 276-285.

ABSTRACT: During the design stage, tire designers have to use trial and error to decide on the design factors in order to satisfy performance requirements, and this involves a great deal of time and expense. Optimization is one of the methods which can be used effectively and efficiently to improve tire performance. There is a lot of literature available about optimal tire contour and structure design. On the other hand, there is little published information available about optimal tread pattern design. In particular, there is little information available about the interaction between optimal tire contour and tread pattern design. In this study, we constructed a tire optimization system in which the design factors of cross-sectional contour and tread pattern shape could both be dealt with as design variables at the same time. This optimization system was then applied to and verified for an actual tire design problem.

KEY WORDS: optimization, tire tread pattern, cross-sectional contour, basis vector method, finite element analysis, computer-aided engineering

Introduction

Structural optimization procedures based on finite element analysis (FEA) have been successfully employed in the tire industry and there are a large number of examples of its use in published papers. In most cases, these reports concern themselves with the determination of the optimized cross-sectional contour of tires—one of the key elements of tire design [*1*–*3*]. However, in the authors' opinion, there is another equally important element of tire design to be considered—the tread pattern. Although the tread pattern affects many of the functionalities of tires such as wear, handling, etc., numerical optimization methods have rarely been used for the design of tread patterns due to the complexity of numerical identification of the design elements involved. As far as we know, only one study of tread block surface optimization using optimality criteria has been carried out [*4*].

The purpose of this study was to construct an algorithm for optimization which could define simultaneously both the design factors of the cross-sectional contour and the design factors of pattern shape for use as design parameters and

[1] Presenter. Toyo Tire & Rubber Co., Ltd., Tire Technical Center, 2-2-13 Fujinoki, Itami, Hyogo, 664-0847, Japan. Electronic mail: y-tanaka@toyo-rubber.co.jp

[2] Toyo Tire & Rubber Co., Ltd., Tire Technical Center, 2-2-13 Fujinoki, Itami, Hyogo, 664-0847, Japan. Electronic mail: ohishi@toyo-rubber.co.jp

could identify mutual relationships between the tread pattern shape and the cross-sectional contour, and the contribution ratio of each design parameter. To achieve this goal, an algorithm for pattern shape optimization was constructed by combining the basis vector method, which is one of the most widely used shape optimization methods [5,6], FEA [7], the response surface method (RSM), a parametric assignment strategy normally used for the design of experiments (DOE) [8,9], and the sequential quadratic programming (SQP) method [10,11]. An algorithm for optimization was then constructed which could define the design variables of the contour design factors and the pattern shape design factors at the same time. These algorithms were then applied to optimization problems involving actual tire design.

Governing Equations

In the finite element solver, nonlinear finite element analysis [7] can be used for analyzing the deformation of structure. Total Lagrangian formulation includes all kinetic nonlinear effects due to large displacements, and large rotations and large strain can be considered.

Numerical Methods

Numerical Method for Structural Optimization

The virtual work done during a displacement variation is equal to the variation of total potential energy. Therefore, the equation of the system may be given as

$$[K]\{u\} = \{f\} \tag{1}$$

Where [K], {u}, and {f} are the stiffness matrix, displacement vector, and nodal force vector, respectively.

The Basis Vector Method for Pattern Parameterization

In general, structural optimization methods are classified as either sizing optimization, shape optimization, or topology optimization. The basis vector method is a typical structural optimization method used for shape optimization. The optimal solution can be easily solved by means of the basis vector method. As such, an expected solution can be obtained, and the shape of the structure does not change. On the other hand, the outline of a structure cannot always be clearly represented, even though the shape of the structure can be drastically changed using topology optimization. Therefore, each numerical method has both advantages and disadvantages.

However, one feature of the basis vector method does enable it to be used

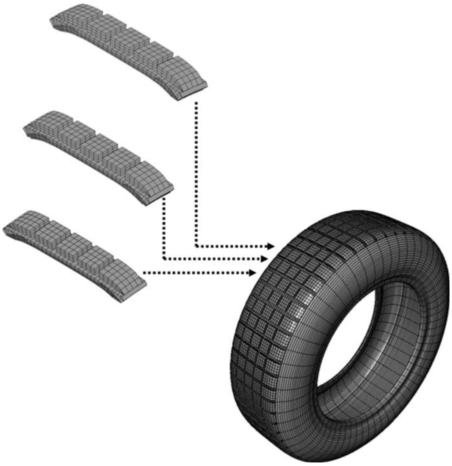


FIG. 1 — *The schematic of the tread pattern.*

to define the outline of a structure as a numerical parameter. If an outline of various structures is prepared, a synthetic shape can be generated using a linear combination of vectors. The desired design can then be expressed in terms of a linear combination of vectors as

$$G = G_0 + \sum x_i (G_i - G_0), \tag{2}$$

where G is the updated shape, G_0 is the initial shape, G_i is the ith basis vector and x_i is the weighting factor. The weighting factor can be defined as a design variable.

In general, it is difficult to represent the tread pattern shape mathematically. The aim of this study was to propose a useful means of designing a tread pattern. The basis vector method was suitable for this purpose because the optimized shape could be solved as the best possible combination of basis vectors.

A modification of the basis vector method was considered in order to expand its usefulness to tire design problems. The basis unit of any tread pattern is repeated many times along the circumference of the tire. Figure 1 shows the layout of the tread pattern. The basic unit of the tread pattern can be defined as the optimized domain. The stress and strain field of the proposed models can be solved using the finite element method, and the objective function can be computed from this result. The tread pattern can be defined as the basis vector shape and Fig. 2 shows the schematic of this shape. In this case, the initial shape, two basis shapes, and weighting factors (x_1, x_2) set at 0.5 were used. Therefore, the synthetic shape could be described using vectors. The key feature of the first

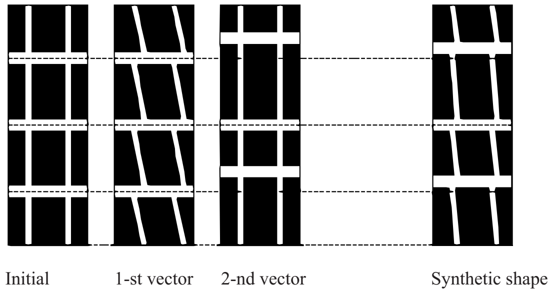


FIG. 2 — *The schematic of the basis vector shape.*

basis vector is the side grooves inclined to the horizontal axis. On the other hand, the key feature of the second basis vector in the first and third groove can be translated in the lateral direction of the tire. Therefore, the synthetic shape can share the features of both the first and second shape.

Numerical Optimization

Optimality criterion methods and advanced numerical optimization methods are typical numerical methods used in the mathematical programming of numerical optimization. However, these methods involve a great deal of computational resources and expense. A nonlinear optimization method based on a method normally used for the design of experiments was adopted in this study in order to reduce computational costs.

Figure 3 shows the computational strategy adopted. First, main factors and

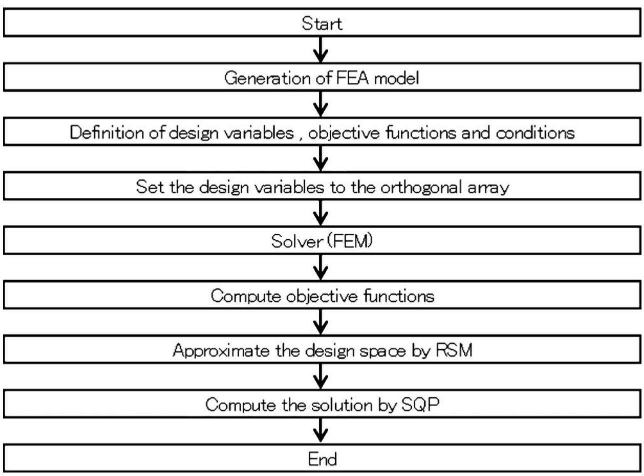


FIG. 3 — *Computational strategy for the optimization process.*

TABLE 1 — *L27 Orthogonal Array.*

		Factor												
		1	2	3	4	5	6	7	8	9	10	11	12	13
Data number	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	2	1	1	1	1	2	2	2	2	2	2	2	2	2
	3	1	1	1	1	3	3	3	3	3	3	3	3	3
	4	1	2	2	2	1	1	1	2	2	2	3	3	3
	5	1	2	2	2	2	2	2	3	3	3	1	1	1
	6	1	2	2	2	3	3	3	1	1	1	2	2	2
	7	1	3	3	3	1	1	1	3	3	3	2	2	2
	8	1	3	3	3	2	2	2	1	1	1	3	3	3
	9	1	3	3	3	3	3	3	2	2	2	1	1	1
	10	2	1	2	3	1	2	3	1	2	3	1	2	3
	11	2	1	2	3	2	3	1	2	3	1	2	3	1
	12	2	1	2	3	3	1	2	3	1	2	3	1	2
	13	2	2	3	1	1	2	3	2	3	1	3	1	2
	14	2	2	3	1	2	3	1	3	1	2	1	2	3
	15	2	2	3	1	3	1	2	1	2	3	2	3	1
	16	2	3	1	2	1	2	3	3	1	2	2	3	1
	17	2	3	1	2	2	3	1	1	2	3	3	1	2
	18	2	3	1	2	3	1	2	2	3	1	1	2	3
	19	3	1	3	2	1	3	2	1	3	2	1	3	2
	20	3	1	3	2	2	1	3	2	1	3	2	1	3
	21	3	1	3	2	3	2	1	3	2	1	3	2	1
	22	3	2	1	3	1	3	2	2	1	3	3	2	1
	23	3	2	1	3	2	1	3	3	2	1	1	3	2
	24	3	2	1	3	3	2	1	1	3	2	2	1	3
	25	3	3	2	1	1	3	2	3	2	1	2	1	3
	26	3	3	2	1	2	1	3	1	3	2	3	2	1
	27	3	3	2	1	3	2	1	2	1	3	1	3	2

interactions are assigned to columns of orthogonal arrays. All the design points of the DOE design matrix are then generated as FEM data. The use of orthogonal arrays for the DOE provides an efficient and effective method for determining the most significant factors and interactions in any given design problem. Table 1 shows the L27 orthogonal array used along with the factor/interaction assignments. The number 27 in the title refers to the number of analyses. Analyses for all design points of the DOE design matrix can be executed using the finite element solver and objective functions can then be computed from each computational result. For example, the L27 array provides for 27 data points. DOE results should be analyzed using analysis of variance in order to identify the influence of individual factors and interactions. A three-level analysis with interactions can be used for the L27 array and response surface models can be

TABLE 2 — *Computational conditions of FEA.*

Size	215/55R17
Pressure [kPa]	230
Vertical load [kN]	3.5
Lateral load [kN]	1.0

constructed using the orthogonal array's design points. The formulation of a response surface can be defined using a second-order approximation function. Design optimization can then be solved using the response surface models. In general, optimization problems can be defined as

Minimize: $f(x)$

Subject to

$$g_i(x) \geq 0 \quad (i = 1,2, \ldots ,k),$$

$$h_j(x) \geq 0 \quad (j = 1,2, \ldots ,m), \tag{3}$$

where $f(x)$ is an objective function, g_i and h_j are design constraint functions, x is a design variable, and k and m are the number of inequality equations. When applied to a tire design problem, the objective function can be defined as the contact pressure variance, the stiffness, and so on, and the constraint condition can be defined as the contact length ratio. The optimal solution can then be obtained using a sequential approximate optimization process, until convergence occurs.

Results and Discussion

Computational Conditions

Table 2 shows the tire size and computational conditions. Figure 4 shows the definition of the cross-sectional contour of the tires and the proposed design vectors. These basic shapes were constructed in order to improve the contact pressure. The effects of the tread pattern design are considered to extend across the width of the sipe, the location of the tread block, and the width of the tread block.

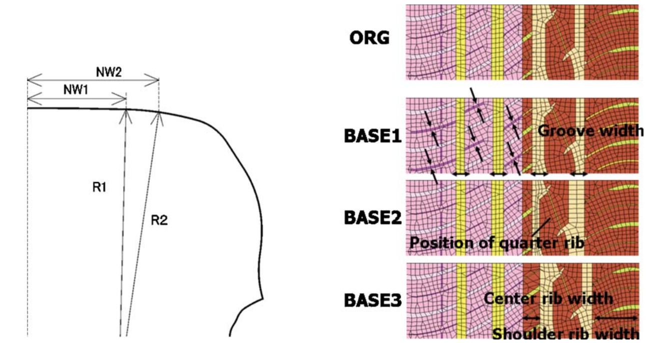
Definitions of terms regarding numerical optimization are as follows:

Optimization method: DOE-based RSM

Orthogonal array: L27

Objective function: Minimize the contact pressure variance

The objective function is defined by the following equation



a) Cross-sectional contour b) Basis shapes

FIG. 4 — *The schematic of design variables used for the optimization process.*

$$f(x) = \sum_{i=1}^{N} \left\{ \frac{S_i}{\sum_{j=1}^{N} S_j} (P_i - \bar{p})^2 \right\}, \tag{4}$$

where P_i is the pressure of ith element, $\bar{p}$ is the mean contact pressure, S_i is the contact area of ith element, and N is the number of elements.

Design variables: the cross-sectional contour of tires and the weighting factors

The ith shape design variable is defined as x_i.

Interactions: Base. 1 and Base. 2, Base. 1 and R2, Base. 2 and R2

For DOE analysis, a three-level experiment with interactions can be used. The L27 array provides for 27 data points. With thirteen columns, design variables can be assigned along with three interactions. The three interactions assigned in this paper are Base. 1-Base. 2, Base. 1-R2 and Base. 2-R2. Main factors, Base. 1, Base. 2, Base. 3, R2 and NW2, can be assigned to the first, second, ninth, fifth, and tenth columns, respectively.

Constraint conditions: Table 3 lists these conditions. In this table, constraint

TABLE 3 — *Constraint conditions.*

Design variable	Row number	Range of design variables
Base. 1	1	0.0 ~ 1.0
Base. 2	2	−0.7 ~ 0.7
Base. 3	9	0.0 ~ 1.2
R2	5	0.8 ~ 1.2
NW2	10	0.9 ~ 1.1

TABLE 4 — *Comparison of the computational results.*

	Initial design value	Optimization value	Significance
Base. 1	0.00	0.80	*
Base. 2	0.00	0.14	**
Base. 3	0.00	0.96	**
R2	1.00	1.13	**
NW2	1.00	0.91	
Base. 1 * Base. 2			*
Base. 1 * R2			
Base. 2 * R2			*

conditions are the weighting factors in Base. 1, Base. 2 and Base. 3. Other constraint conditions are the normalized values for each dimensional value of the original tire.

Computational Results

Using the algorithm proposed above, a numerical solution could be solved to reduce the contact pressure variance in comparison with the pressure variance of the initial tire. Table 4 shows the computational results for each design parameter. Figure 5 shows the comparison of the objective function. From this result, it can be seen that the objective function could be reduced by about 9% in comparison with the initial tire design. Figure 6 shows a comparison of the contact pressure distributions. From this result, it can be seen that the contact pressure at the center and outer regions could be reduced drastically. A notice-

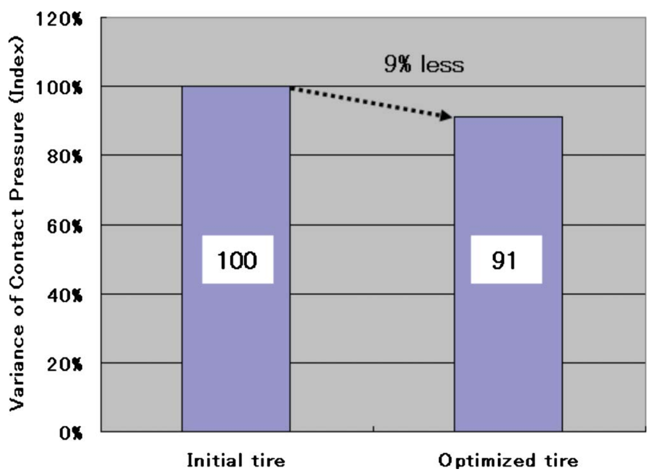
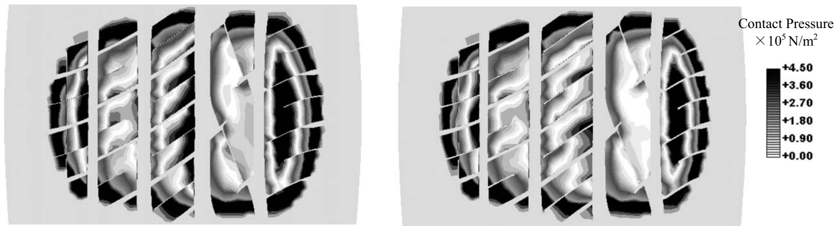


FIG. 5 — *Comparison of the objective function.*



a) Initial tire b) Optimized tire

FIG. 6 — *Comparison of the contact pressure distributions.*

able feature of this solution for the tread pattern is that the center rib width has been increased. On the other hand, the shoulder rib width has been reduced. Another noticeable feature of this numerical solution for the cross-sectional contour of the tire is the level contour in the outer region. The contact pressure in the center region of the initial tire is larger than the outer region. Therefore, the optimized solution has converged to the wide block in the center region.

Based on this analysis, R2, Base. 2 and Base. 3 can be considered significant variables at the 1% significance level whereas Base. 1, interaction with both Base. 1 and Base. 2 and interaction with both Base. 2 and R2 can be considered significant variables at the 5% significance level.

If the contact pressure variance is reduced, the irregular wear and the loss of grip in cornering conditions can be improved, as shown in Ref. [1]. If the tire contour is modified, the contact pressure variance can be reduced in comparison with the initial tire contour. So, evaluations of dry and wet handling at the proving ground can both be improved by adoption of the optimum contour. The optimum contour also shows uniform wear because this contour has a more uniform pressure distribution. Consequently, evaluations of dry handling, wet handling and wear are in good agreement with the contact pressure variance. Therefore, the proposed numerical method can be considered a valid means of predicting the evaluation of these performance indicators.

In this paper, the interaction with both Base. 2 and R2 is significant at the 1% significance level. If the value of R2 is changed, the effect of Base. 2 is different at each value of R2. Therefore, it is important to consider the interaction between the tread pattern variables and the cross-sectional contour variables involved in the tire design.

Conclusions

(1) The proposed algorithm for numerical optimization was constructed in order to define simultaneously the design variables of tread pattern and

the cross-sectional contour. The basis vector method was applied to tread pattern optimization because it is a suitable means for defining the shape of a tread pattern.

(2) The proposed algorithm was applied to a tire design problem, and a numerical solution was obtained that could reduce the contact pressure variance.

(3) This shows that it is important to consider the interaction between tread pattern variables and the cross-sectional contour variables at the tire design stage.

[1] Nakajima, Y., Kadowaki, H., Kamegawa, T., and Ueno, K., "Application of a Neural Network for the Optimization of Tire Design," *Tire Sci. Technol.*, Vol. 27, 1999, pp. 62–83.

[2] Koide, M., Heguri, H., Kamegawa, T., Nakajima, Y., and Ogawa, H., "Optimization for Motorcycle Tire Using Explicit FEM," *Tire Sci. Technol.*, Vol. 29, 2001, pp. 230–243.

[3] Cho, J. R., Jung, H. S., Kim, N. J., and Kim, K. W., "Application of STOM to the Optimal Tire Contour Design by Introducing the Aspiration-level Indicator," *Tire Sci. Technol.*, Vol. 30, 2002, pp. 265–288.

[4] Nakajima, Y., Takahashi, F., and Abe, A., "Surface Shape Optimization of Tire Pattern by Optimality Criteria," *Tire Sci. Technol.*, Vol. 31, 2003, pp. 2–18.

[5] Leila, J. P., and Watson, B. C., "Automatic Generation of Basis Vectors for Shape Optimization in the Genesis Program," *American Institute of Aeronautics and Astronautics*, 1998–4852.

[6] Zhang, J., "Modified Thermal Load Approach for Automatic Generation of Basis Vectors in Shape Optimization," *American Institute of Aeronautics and Astronautics*, 1994–4292-CP.

[7] Bathe, K. J., *Finite Element Procedures* (Prentice-Hall, Englewood Cliffs, NJ, 1996).

[8] *iSIGHT Version 9.0 Reference Guide*, Engineous Software, 2004.

[9] Kodiyalam, S., Lin, J. S., and Wujek, B. A., "Design of Experiments Based Response Surface Models for Design Optimization," *The 39th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference*, 1998.

[10] Gill, P. E., Murray, W., Saunders, M. A., and Wright, M. H., "Procedures for Optimization Problems with a Mixture of Bounds and General Linear Constraints," *ACM Trans. Math. Softw.*, Vol. 10, 1984, pp. 282–298.

[11] Gill, P. E., Murray, W., and Wright, M. H., *Numerical Linear Algebra and Optimization* (Addison-Wesley, Reading, MA, 1991), Vol. 1.

Presented at the twenty-seventh annual meeting of The Tire Society, Akron, Ohio, September 15–16, 2008.