Mathematical Methods for Engineers and Scientists 3

K.T. Tang

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Fourier Analysis, Partial Differential Equations and Variational Methods

With 79 Figures and 4 Tables

Springer

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Preface

For some 30 years, I have taught two "Mathematical Physics" courses. One of them was previously named "Engineering Analysis." There are several textbooks of unquestionable merit for such courses, but I could not find one that fitted our needs. It seemed to me that students might have an easier time if some changes were made in these books. I ended up using class notes. Actually, I felt the same about my own notes, so they got changed again and again. Throughout the years, many students and colleagues have urged me to publish them. I resisted until now, because the topics were not new and I was not sure that my way of presenting them was really much better than others. In recent years, some former students came back to tell me that they still found my notes useful and looked at them from time to time. The fact that they always singled out these courses, among many others I have taught, made me think that besides being kind, they might even mean it. Perhaps, it is worthwhile to share these notes with a wider audience.

It took far more work than expected to transcribe the lecture notes into printed pages. The notes were written in an abbreviated way without much explanation between any two equations, because I was supposed to supply the missing links in person. How much detail I would go into depended on the reaction of the students. Now without them in front of me, I had to decide the appropriate amount of derivation to be included. I chose to err on the side of too much detail rather than too little. As a result, the derivation does not look very elegant, but I also hope it does not leave any gap in students' comprehension.

Precisely stated and elegantly proved theorems looked great to me when I was a young faculty member. But in the later years, I found that elegance in the eyes of the teacher might be stumbling blocks for students. Now I am convinced that before a student can use a mathematical theorem with confidence, he or she must first develop an intuitive feeling. The most effective way to do that is to follow a sufficient number of examples.

This book is written for students who want to learn but need a firm hand-holding. I hope they will find the book readable and easy to learn from.

Learning, as always, has to be done by the student herself or himself. No one can acquire mathematical skill without doing problems, the more the better. However, realistically students have a finite amount of time. They will be overwhelmed if problems are too numerous, and frustrated if problems are too difficult. A common practice in textbooks is to list a large number of problems and let the instructor to choose a few for assignments. It seems to me that is not a confidence building strategy. A self-learning person would not know what to choose. Therefore a moderate number of not overly difficult problems, with answers, are selected at the end of each chapter. Hopefully after the student has successfully solved all of them, he will be encouraged to seek more challenging ones. There are plenty of problems in other books. Of course, an instructor can always assign more problems at levels suitable to the class.

Professor I.I. Rabi used to say "All textbooks are written with the principle of least astonishment." Well, there is a good reason for that. After all, textbooks are supposed to explain away the mysteries and make the profound obvious. This book is no exception. Nevertheless, I still hope the reader will find something in this book exciting.

This set of books is written in the spirit of what Sommerfeld called "physical mathematics." For example, instead of studying the properties of hyperbolic, parabolic, and elliptic partial differential equations, materials on partial differential equations are organized around wave, diffusion and Laplace equations. Physical problems are used as the framework for various mathematical techniques to hang together, rather than as just examples for mathematical theories. In order not to sacrifice the underlying mathematical concepts, these materials are preceded by a chapter on Sturm–Livouville theory in infinite dimensional vector space. It is author's experience that this approach not only stimulates students' intuitive thinking but also increase their confidence in using mathematical tools.

These books are dedicated to my students. I want to thank my A and B students, their diligence and enthusiasm have made teaching enjoyable and worthwhile. I want to thank my C and D students, their difficulties and mistakes made me search for better explanations.

I want to thank Brad Oraw for drawing many figures in this book and Mathew Hacker for helping me to typeset the manuscript.

I want to express my deepest gratitude to Professor S.H. Patil, Indian Institute of Technology, Bombay. He has read the entire manuscript and provided many excellent suggestions. He has also checked the equations and the problems and corrected numerous errors.

The responsibility for remaining errors is, of course, entirely mine. I will greatly appreciate if they are brought to my attention.

Tacoma, Washington *K.T. Tang*
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