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This bibliography includes the references cited in the text and a few other books and tables that might be useful.

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